The Mermin-Peres Magic Square Game

2 Bell-pairs = 4 qubits: A1, A2 for Alice and B1, B2 for Bob
and one common 3 x 3 measurement operator square

	Column 1	Column 2	Column 3
Row 1	I⊗Z	Z⊗I	Z⊗Z
Row 2	X⊗I	I⊗X	X⊗X
Row 3	-X⊗Z	-Z⊗X	Y⊗Y

Alice measures both A1, A2

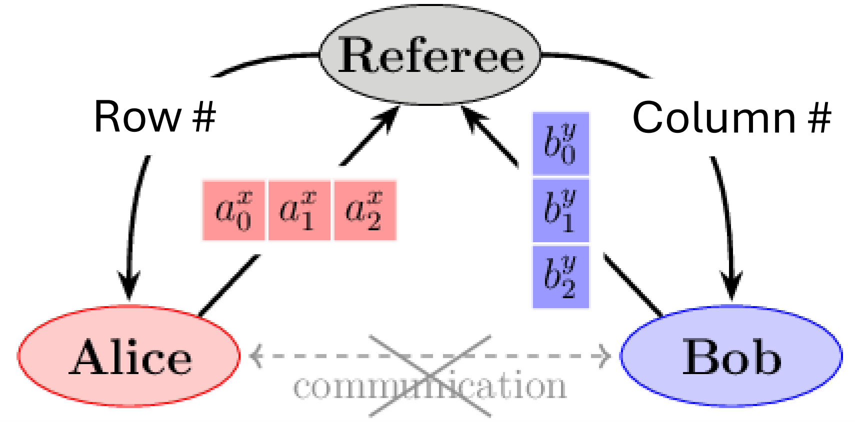
Bob measures both B1, B2

No communication once measurement begins

Abstract

The Mermin–Peres magic square game is a striking, minimal demonstration that quantum mechanics permits coordination between separated parties that no classical strategy can match. Two players, Alice and Bob, answer row and column queries about a 3×3 grid without communicating during play. A simple parity argument shows that any classical, pre-agreed table must fail on at least one of the nine possible queries — capping classical success at 8 out of 9. Sharing two entangled qubit pairs, Alice and Bob can win every single round, with certainty. This walkthrough builds the game from its classical impossibility, through the entangled resource and the nine quantum measurement operators that realize it, to the operator-algebra "magic" — two deliberately sign-flipped cells — that makes the impossible achievable, and closes with the game's connection to the Kochen–Specker theorem on quantum contextuality.

The Mermin-Peres Magic Square Game



one common 3 x 3 +/- square (only one of several possibilities):

+	+	+	$\prod$ = +
+	+	+	$\prod$ = +
-	-	+/-	$\prod$ = +
$\prod$ = -	$\prod$ = -	$\prod$ = -	

Alice and Bob will lose this round!

All other rounds they will win

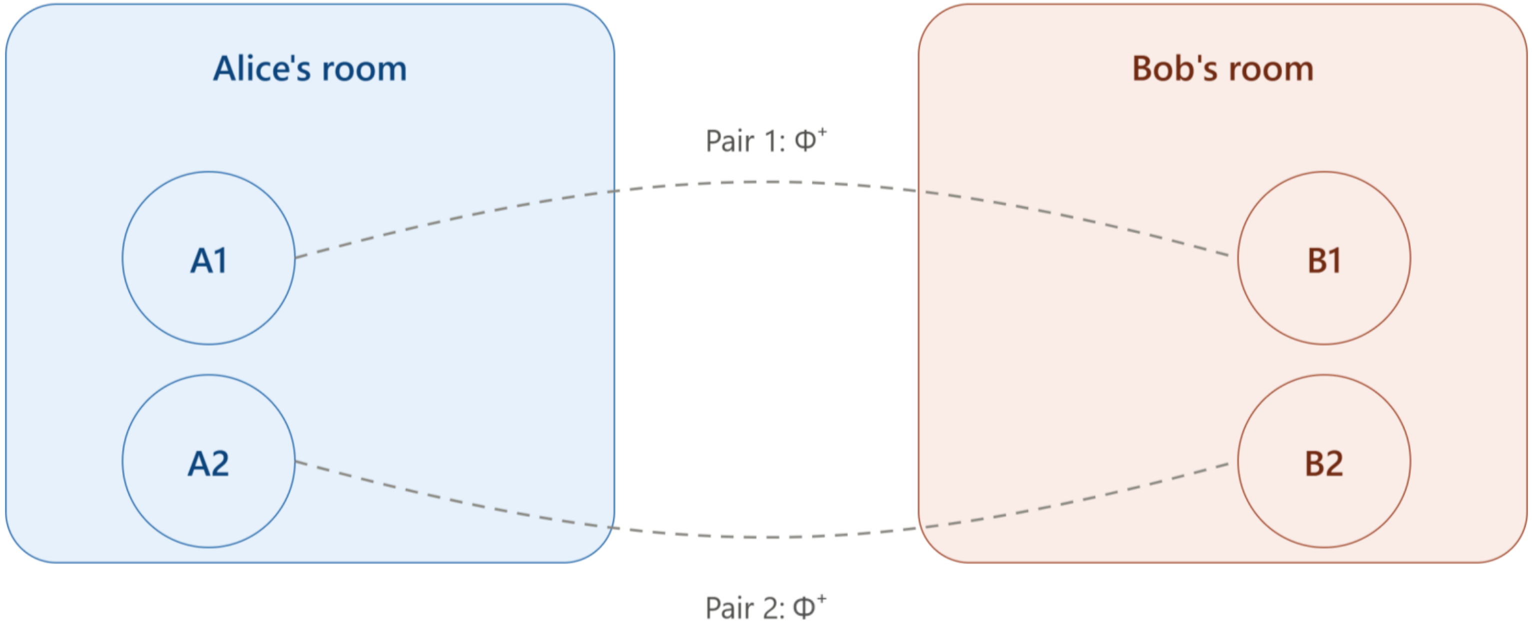
Introduction

Can two parties who cannot communicate still act as if they had agreed on an impossible answer key? Classically, no — and the reason is a simple, checkable fact about parity: no assignment of plus and minus signs to a 3×3 grid can make every row multiply to +1 while every column multiplies to −1. Any strategy built in advance, however cleverly randomized, must fail on at least one of the nine row–column combinations a referee could ask about.

The Mermin–Peres magic square game turns this impossibility into a test. A referee sends Alice a row number and Bob a column number, chosen independently and unpredictably. Each must answer with three values for their line, satisfying their line's required product, and the value they report for the cell they share must agree. What follows shows how two shared entangled qubit pairs let Alice and Bob pass this test in every round — not most rounds, all of them — without violating the parity argument that defeats every classical strategy.

The Mermin-Peres Magic Square Game

2 Bell-pairs = 4 qubits: A1, A2 for Alice and B1, B2 for Bob



Alice measures both A1, A2

Bob measures both B1, B2

No communication once measurement begins

The Mermin-Peres Magic Square Game

Instead of one common 3 x 3 +/- square,
one common 3 x 3 measurement operator square and **2 entangled Bell pairs**

(This is only one example square — not the only possible arrangement of nine operators)

	Column 1	Column 2	Column 3
Row 1	$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
Row 2	$X \otimes I$	$I \otimes X$	$X \otimes X$
Row 3	$-X \otimes Z$	$-Z \otimes X$	$Y \otimes Y$

The Mermin–Peres Magic Square Game

Alice's room — row 2 = (X⊗I, I⊗X, X⊗X)

Measure X⊗I
X on A1, I on A2
outcome $a_0 = +1$

→

Measure I⊗X
I on A1, X on A2
outcome $a_1 = +1$

→

Measure X⊗X
X on A1, X on A2
outcome $a_2 = +1$

All three commute — same underlying state, compatible measurements

Alice's row 2 answer: (+1, +1, +1)
product $a_0 \cdot a_1 \cdot a_2 = +1$ — row parity satisfied

Sent to referee as three classical bits — no further quantum step

The Mermin-Peres Magic Square Game

These are three separate measurements on the same two qubits, not three independent systems. Alice only has A1 and A2 — she performs three different two-qubit measurements on this one pair, one after another: X on A1 combined with "measure nothing" (identity I) on A2, then the reverse, then X on both simultaneously. Each of these three measurements yields its own ±1 value.

Why this works at all without destroying the qubits: the three operators $X \otimes I$, $I \otimes X$, and $X \otimes X$ commute with one another — they don't disturb each other when measured in succession (unlike, say, X and Z on the same qubit, which don't commute and would "erase" each other). This is no accident; it is precisely the property Mermin and Peres built into the square by construction — only commuting operators per row or column make the measurement well-defined in the first place.

That all three come out +1 is, in this example, a matter of the specific state preparation — with a different sign combination (say, +1, −1, −1) the product would still be guaranteed +1, because the algebraic structure of the operators forces it ($X \otimes I \cdot I \otimes X \cdot X \otimes X = X^2 \otimes X^2 = I \otimes I$, eigenvalue +1). The row parity is therefore not luck but law — Bob has the same guarantee for his column, only with product −1.

The Mermin–Peres Magic Square Game

Bob's room — column 3 = $(Z\otimes Z, X\otimes X, Y\otimes Y)$

Measure $Z\otimes Z$
Z on B1, Z on B2
outcome $b_0 = +1$

→

Measure $X\otimes X$
X on B1, X on B2
outcome $b_1 = +1$

→

Measure $Y\otimes Y$
Y on B1, Y on B2
outcome $b_2 = -1$

All three commute — same underlying state, compatible measurements

Bob's column 3 answer: $(+1, +1, -1)$ — product $= -1$ *)

Shared cell: row 2 × column 3 = $X\otimes X$

Alice's third value $a_2 = +1$

Bob's second value $b_1 = +1$

$a_2 = b_1 = +1$ — they match, round won

The Mermin-Peres Magic Square Game

Alice's room — row 3 = ($-X\otimes Z$, $-Z\otimes X$, $Y\otimes Y$)

Measure $-(X\otimes Z)$
X on A1, Z on A2, sign flip
outcome $a_0 = -1$

→ Measure $-(Z\otimes X)$
Z on A1, X on A2, sign flip
outcome $a_1 = +1$

→ Measure $Y\otimes Y$
Y on A1, Y on A2, no sign
outcome $a_2 = -1$

Alice's row 3 answer: $(-1, +1, -1)$ — product $= +1$

Bob's room — column 3 = ($Z\otimes Z$, $X\otimes X$, $Y\otimes Y$)

Measure $Z\otimes Z$
Z on B1, Z on B2
outcome $b_0 = +1$

→ Measure $X\otimes X$
X on B1, X on B2
outcome $b_1 = +1$

→ Measure $Y\otimes Y$
Y on B1, Y on B2
outcome $b_2 = -1$

Bob's column 3 answer: $(+1, +1, -1)$ — product $= -1$ *)

Shared cell $Y\otimes Y$: $a_2 = b_2 = -1$ — they match, round won

*) Bob's column-3 values (+1, +1, −1) are repeated identically in panels 9 and 10 purely for simplicity of illustration. In reality, each panel represents an independent round with freshly measured entanglement — Bob's individual outcomes could differ between them. Only the column product (−1) and the match at the shared cell are guaranteed.

The Mermin-Peres Magic Square Game

The shared cell at row 3 / column 3 is $Y \otimes Y$, but the sign subtlety that makes the square work doesn't live there — it lives in the *other two* cells of row 3. The operators are $-(X \otimes Z)$, $-(Z \otimes X)$, and $Y \otimes Y$: the first two carry an explicit minus sign as part of the physical observable itself, not merely as a matter of which eigenvalue happens to come out. Without those two minus signs, all three rows would still multiply to +1, but only column 3 would correctly multiply to −1 — columns 1 and 2 would come out to +1, breaking the very structure the game depends on. Only with both minus signs in place do all three columns multiply to −1 while all three rows still multiply to +1. That is the actual "magic" in the Mermin-Peres square: nine operators, built from an odd number of sign-flipped cells, that make an otherwise impossible classical parity pattern achievable quantum mechanically. $Y \otimes Y$ itself carries no special sign; its role is simply to complete row 3 and column 3 with a mutually commuting operator.

The illustrative values ($a_0 = -1$, $a_1 = +1$, $a_2 = -1$, and $b_2 = -1$ for the shared cell) are an example assignment, not derived from the operator algebra. The exact eigenvalues depend on the specific four-qubit state chosen; the numbers are chosen only to be internally consistent (row product +1, column product −1, shared cell equal) — not to correspond to an actual measurement on a specific state.

The Mermin–Peres Magic Square Game

The referee never checks consistency across rounds — only within one.

Each round of the magic square game is an independent event: a fresh pair of Bell pairs, a freshly chosen row and column, and a check limited strictly to that single round — row product +1, column product −1, shared cell matching. The referee never compares answers from different rounds, because the rules never require it.

This isn't a loophole in the game — it's the reason the quantum strategy works at all. The quantum protocol doesn't produce a hidden, consistent table of nine fixed values that survives scrutiny across repeated play; individual outcomes are genuinely random from round to round, constrained only by the row/column parity. Demanding table-wide consistency across all nine cells simultaneously is a different, stricter question — one answered by the Kochen–Specker theorem, which shows no such fixed table can exist, quantum or classical. The magic square game sidesteps that impossibility rather than defeating it: by never requiring cross-round consistency, it never puts that impossibility to the test.

Conclusion

The magic square game distills several deep quantum ideas into a single checkable rule. Classically, no fixed 3×3 table can satisfy the required row and column parities at once — every classical strategy loses at least one round in nine. Quantum mechanically, two shared entangled pairs and nine carefully chosen measurement operators satisfy both parities in every round, with certainty.

The mechanism is precise, not mysterious: two of the nine operators must carry an explicit minus sign — not the operator at the shared cell, but the other two entries sharing its row — and without both signs in place the construction collapses back into the classical impossibility. What resolves the apparent paradox further is that the referee never checks consistency across rounds, only within one: individual outcomes are genuinely random from round to round, and only the parity constraints are guaranteed. Demanding a single, round-independent table of nine fixed values is a stricter question altogether — one the Kochen–Specker theorem answers with a flat no, for any theory, quantum or classical.

The game is therefore best understood not as a loophole but as a sharp illustration of quantum contextuality: measurement outcomes depend on which compatible set of operators they are measured alongside, and no assignment of values existing independently of that context can reproduce what entanglement delivers.

Circuit Implementations - The Mermin–Peres Magic Square Game in Qiskit